Spring 2025 Exam 1

Department of Mechanical Engineering Technology

Time: 75 minutes

This exam consists of 3 pages (including this page).

Do NOT open this exam until you are instructed to do so.

When you are finished, turn in all your materials to the instructor:

- This exam
- The equation sheet
- Your crib sheet

Section 1: Multiple Choice (2 points each, 20 points total)

1. Which of the following statements about factors of safety is *true*?
   - O Higher factors of safety always result in more cost-effective designs.
   - O A lower factor of safety should be used for brittle materials compared to ductile materials.
   - ● The factor of safety accounts for uncertainties in loading, material properties, and manufacturing processes.
   - O A factor of safety is only necessary for cyclic loading conditions.
2. According to the Distortion Energy Theory, failure occurs when:
   - ● The von Mises stress exceeds the yield strength
   - O The maximum normal stress exceeds the ultimate tensile strength
   - O The material reaches its proportional limit
   - O The maximum principal strain exceeds a critical value
3. Which of the following failure theories is most appropriate for predicting failure in brittle materials?
   - O Maximum Shear Stress (MSS) Criterion
   - O Distortion Energy (DE) Criterion
   - ● Modified Mohr (MM) Criterion
   - O Von Mises Criterion
4. The Maximum Shear Stress (MSS) theory (also known as the Tresca criterion) is best suited for predicting failure in:
   - O Brittle materials
   - ● Ductile materials
   - O Materials that fail due to creep
   - O Materials under fatigue loading
5. Which type of stress is dominant in a beam subjected to pure bending?
   - O Shear stress
   - O Torsional stress
   - ● Normal stress
   - O Hoop stress
6. When analyzing a circular shaft subjected to torsion, the maximum shear stress occurs:
   - O Along the centerline of the shaft
   - ● On the outer surface of the shaft
   - O Uniformly throughout the cross-section
   - O At a 45° angle from the axis of loading
7. The neutral axis of a beam under bending is:
   - O The line along which the bending stress is maximum
   - ● The axis that experiences zero normal stress
   - O The axis where the shear stress is maximum
   - O Always located at the centroid of the cross-section
8. The shear stress distribution in a rectangular beam under transverse loading is:
   - O Maximum at the top and bottom fibers

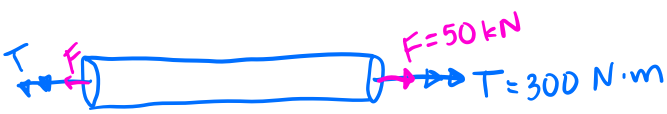
- O Uniform throughout the beam
- O Zero at the centroid and maximum at the outer fibers
- ● Maximum at the centroid and zero at the outer fibers

9. The fracture toughness K_{IC} of a material:
   - O Decreases with increasing crack size
   - ● Is the material's resistance to crack propagation
   - O Depends only on the yield strength of the material
   - O Is higher in brittle materials than in ductile materials
10. The Mode I fracture mechanism corresponds to:
    - O Shear loading
    - O Torsional failure
    - ● ~~Torsional failure~~ TENSILE CRACK OPENING
    - O Bending-induced fracture

Section 2: Problem-Solving (10 points each, 30 points total)

11. A solid circular shaft with a diameter of 40 mm is subjected to an axial force of 50 kN and a torsional moment of 300 N·m. The shaft is made of a brittle material with an ultimate tensile strength of 120 MPa and an ultimate compressive strength of 350 MPa. Determine whether the shaft will fail using:
    a) Modified-Mohr theory
    b) Brittle Coulomb-Mohr theory

    Show all calculations.



AXIAL STRESS DUE TO F IS UNIFORM ACROSS THE CROSS-SECTION

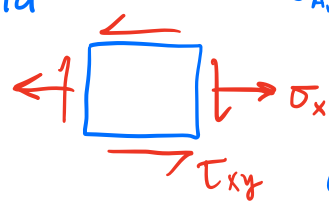
SHEAR STRESS DUE TO T IS MAX @ OUTER SURFACE

↳ PICK ANY POINT ON OUTER SURFACE TO ANALYZE

$\sigma_x = \frac{F}{A} = \frac{50\text{ kN}}{\pi(0.02\text{ m})^2} = 39.8\text{ MPa}$

$\tau_{xy} = \frac{Tr}{J} = \frac{(300\text{ N·m})(0.02\text{ m})}{2.51\times10^{-7}\text{ m}^4} = 23.9\text{ MPa}$

$J = \frac{\pi r^4}{2} = \frac{\pi(0.02\text{ m})^4}{2} = 2.51\times10^{-7}\text{ m}^4$



$\sigma_{A,B} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= 19.9 \pm \sqrt{396 + 571}$

$= 19.9 \pm 31.1\text{ MPa}$

$\sigma_A = 51\text{ MPa}, \quad \sigma_B = -11.2\text{ MPa}$

(a) $\sigma_A \geq 0 \geq \sigma_B$ AND $|\sigma_A| \geq |\sigma_B|$

$n = \frac{S_{ut}}{\sigma_A} = \frac{120\text{ MPa}}{51\text{ MPa}}$

$\boxed{n = 2.4}$

(b) $\sigma_A \geq 0 \geq \sigma_B$

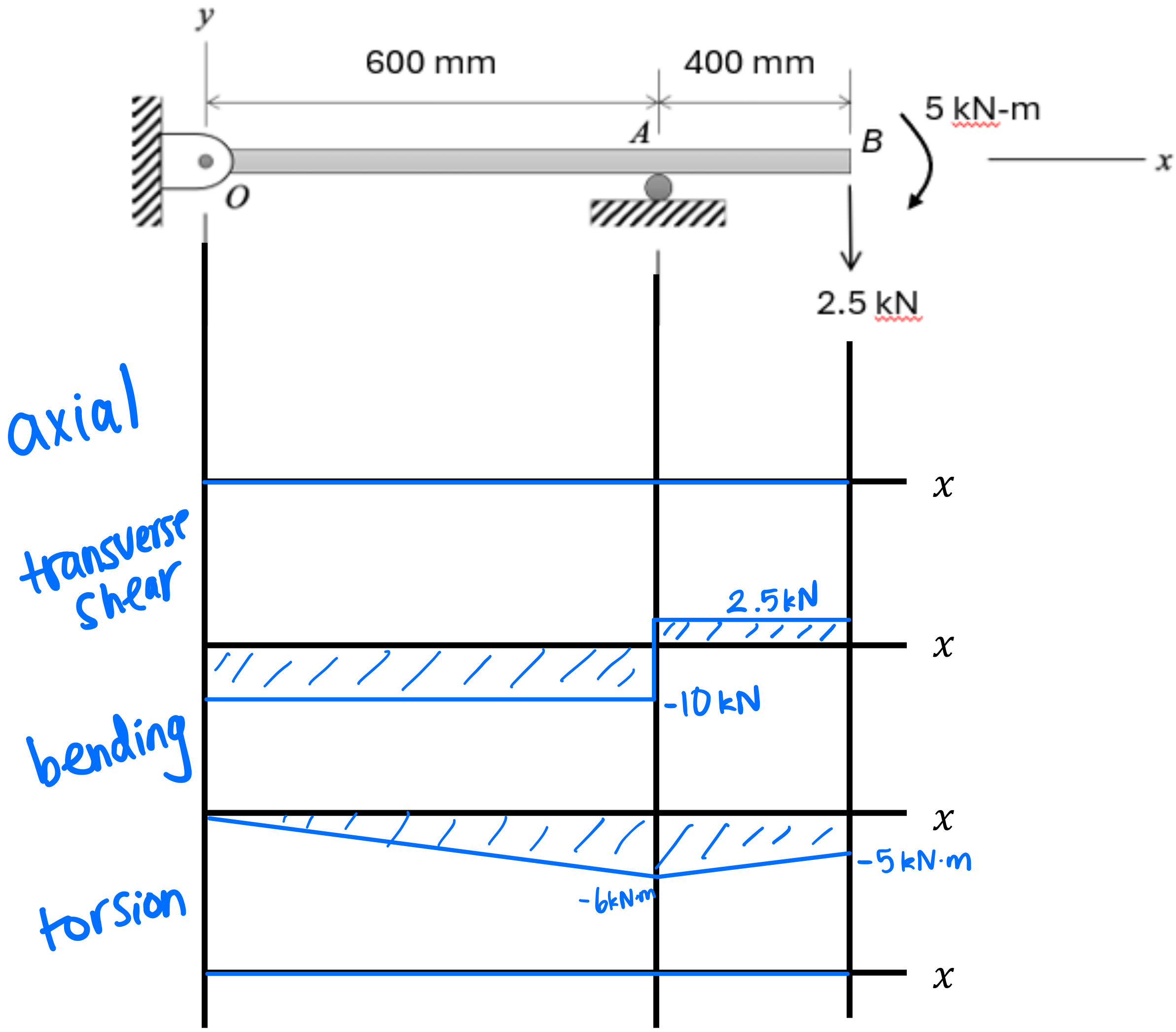
$n = \left(\frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}}\right)^{-1} = \left(\frac{51}{120} - \frac{-11.2}{350}\right)^{-1}$

$\boxed{n = 2.2}$

12. (40 points total) Rod *OAB* has length of 1 m and a diameter of 30 mm.

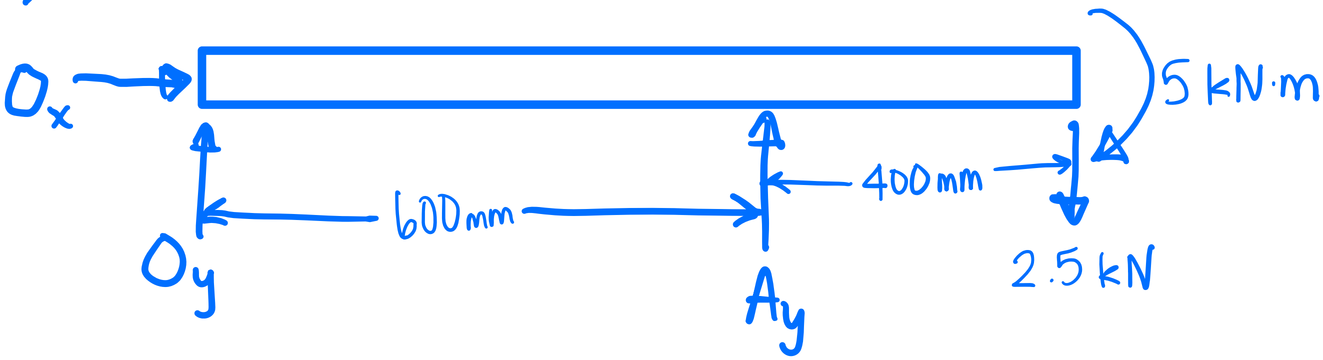
The rod is supported by a pin joint at *O* and by a roller at *A*, which is 600 mm from *O*. A transverse load of 2.5 kN and a bending moment of 5 kN-m are applied at *B*.
The rod is made of a ductile material with yield strength of 250 MPa.
Determine the following:

c) (5 points) Solve for the reactions at *O* and *A*.
d) (10 points) Sketch and label diagrams of the internal loads on the axes provided.
e) (5 points) Identify the critical cross-section of rod *OAB*.
f) (5 points) Identify the critical element on the cross-section identified in part (c). You may use the attached Combined Stress Analysis Worksheet to aid your analysis.
g) (5 points) Show the state of stress on a stress element for the critical element.
h) (10 points) The factor of safety for the critical element using both the distortion energy (DE) and maximum shear stress (MSS) failure theories. If needed, axes to draw Mohr's circle are provided on the next page.



[Problem 12 continued]

(a)



$\Sigma F_x = 0 \Rightarrow$ $\boxed{O_x = 0}$

$\Sigma M_o = 0 \Rightarrow A_y(0.6\,m) - 2.5\,kN(1\,m) - 5\,kN \cdot m = 0$

$$A_y = \frac{7.5\,kN \cdot m}{0.6\,m} \Rightarrow \boxed{A_y = 12.5\,kN}$$

$\Sigma F_y = 0 \Rightarrow O_y + A_y - 2.5\,kN = 0 \Rightarrow \boxed{O_y = -10\,kN}$

(b) SEE ABOVE PAGE.

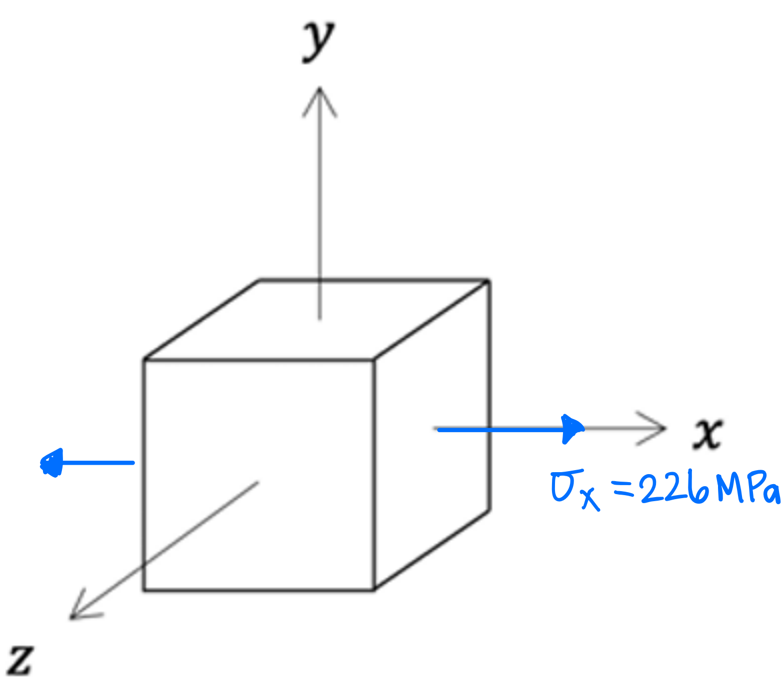
(c) THE CRITICAL CROSS-SECTION IS JUST TO THE RIGHT OF A. (MAX TRANS. SHEAR & BENDING)

(d) SEE ATTACHED WORKSHEET. CRITICAL ELEMENT IS LOCATED ON TOP OF BEAM.

(e) SEE NEXT PAGE

[Problem 12 continued]

Stress element for the critical element:



(f) DE: $\sigma' = 226 MPa$

$n = \frac{S_y}{\sigma'} = \frac{250 MPa}{226 MPa}$

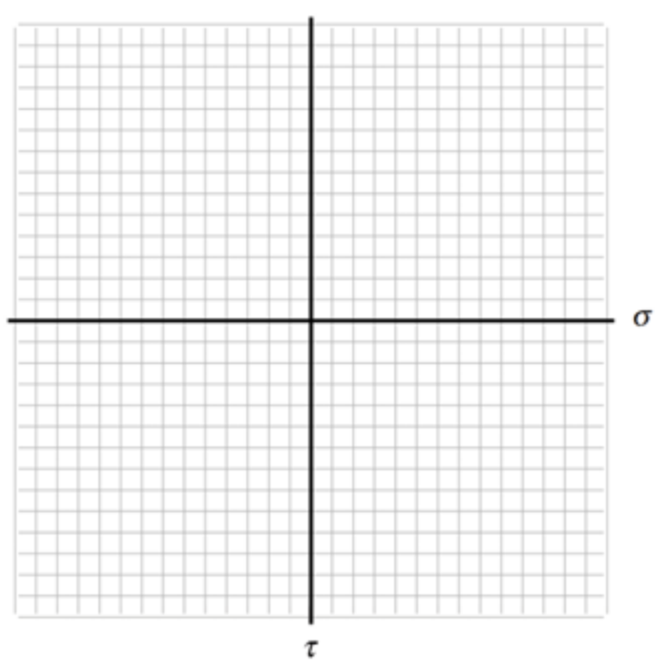
$n = 1.1$

MSS: $n = \frac{S_y}{\sigma_1 - \sigma_3}$

$n = \frac{250 MPa}{226 MPa}$

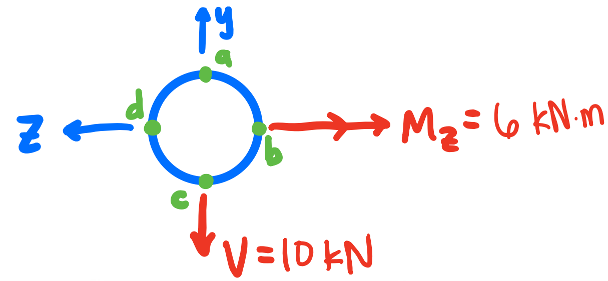
$n = 1.1$

Axes to draw Mohr's circle:



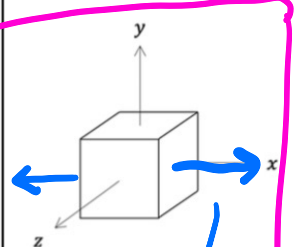
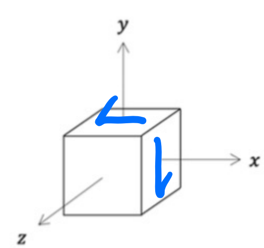
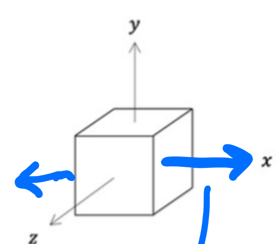
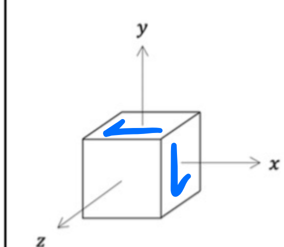
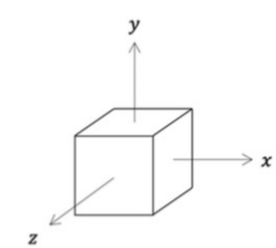
In this box,

- Draw the critical cross-section
- Identify and label the potential locations for the critical element(s) (e.g. top, bottom, left, right, and center)



$I = \frac{\pi r^4}{4} = \frac{\pi(0.015m)^4}{4}$

$I = 3.98\times10^{-8}\,m^4$

Potential location of critical element		a	b	c	d	
Internal load	Axial	none →				
	Torsion	none →				
	Transverse shear	0	$\tau_{xy} = \frac{4V}{3A} = \frac{4(10kN)}{3\pi(0.015m)^2} = 18.9\,MPa$	0	$\tau_{xy} = \frac{4V}{3A} = 18.9\,MPa$	
	Bending	$\sigma_x = \frac{M_z c}{I}$ (tension)	0	$\sigma_x = \frac{-M_z c}{I}$ (compression)	0	
Stress element			 $\tau_{xy} = 18.9\,MPa$		 $\tau_{xy} = 18.9\,MPa$	

$\sigma_x = \frac{(6\,kN\cdot m)(0.015m)}{3.98\times10^{-7}\,m^4}$

$\sigma_x = 226\,MPa$

$\sigma_x = -226\,MPa$